

de Broglie and Heisenberg

Objectives:

Students will understand that both light and matter have wave properties, where the wavelength is related to the momentum through the de Broglie equation $\lambda = h/p$.

Prior Knowledge:

- The photon is a particle of light that has both wave and particle-like properties, depending on the experiment performed (i.e., diffraction of light can only be explained by wave properties while the photoelectric effect can only be explained particle properties). Wave particle duality (WPD): all quantum particles exhibit both wave and particle-like properties.
- Each individual photon has its own energy given by the equation $E = hf$ (h being Planck's constant = 6.63×10^{-34} J•s).
- Students will need to have knowledge of basic wave characteristics such as being able to identify frequency, wavelength and wave speed.
- Students need a basic understanding of constructive and destructive interference when two waves occupy the same space.
- Lasers produce coherent monochromatic light which means all the photons coming out of the laser have the same wavelength and are in phase.
- While not fully necessary for success in this lesson, a basic understanding of the difference between classical and quantum scales and a baseline understanding of Planck's constant is helpful.

NGSS: HS-PS4-3 Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described either by a wave model or a particle model, and that for some situations one model is more useful than the other.

Overview and Background: In the macro world, through classical mechanics, measurements can be made with no theoretical limit on prediction, and deterministic predictions can be made about a future moment in time of a system if the initial conditions of that system are known. For instance, near the surface of Earth when a baseball is thrown into the air, an observer can make very accurate predictions about the velocity of the ball and the position of the ball while it moves through the air if they are given the initial position and momentum of the ball. Someone can also make very accurate measurements of how systems will change with time and other inputs such as external forces. On the quantum scale this is not true because of the nature of wave mechanics. This module, *de Broglie and Heisenberg*, builds on prior modules so students can understand the fundamental idea of the Heisenberg Uncertainty Principle (HUP).

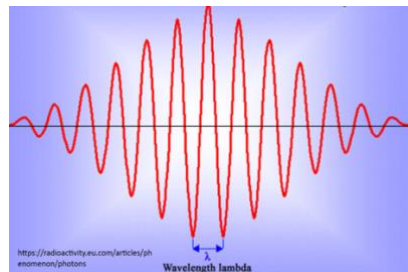
Taking a photo of a speeding baseball is a classical example of how to visualize HUP. If a fast shutter speed is used, the motion is frozen so one knows where the ball is, but no idea how fast it was moving. However, if a slow shutter speed is used, the image is blurry and the streak (smear) allows a determination of the speed and direction, but the exact location of the ball cannot be determined. Therefore, one cannot get a clear picture of both at the same time.

The basic premise of HUP is that position and momentum cannot be known perfectly at the same time and wave packets mathematically prove this limitation. Wave packets are used in

quantum physics to describe the probability of finding a particle in a specific location. A wave packet is a short, localized burst of wave energy that travels as a single unit. It is created by overlapping multiple individual waves with slightly different frequencies and wavelengths that interfere with each other. This interference is typically constructive in one small area and destructively everywhere else. A characteristic of a wave packet is that it looks like a single moving pulse (i.e. localized) which travels at a specific speed. Depending on the medium, the packet may either maintain its shape as it travels or spread out over time (i.e. dispersion). The attribute of the wavefunction that allows the position of a quantum particle to be spread out in space as it moves creates a scenario where quantum particles can “tunnel” through barriers forbidden by classical physics.

To make a wave look like a particle at the same time, one needs to make a packet of waves, as seen in the figure (<https://radioactivity.eu.com/articles/phenomenon/photons>).

To make such a wave packet one needs to combine waves with different wavelengths, and each wavelength has a different momentum. This means any wave packet will have many different momenta so there is uncertainty in the exact momentum of the packet. The tighter in position the packet, more different wavelengths are required which means a tighter wave packet will have greater range of momenta and greater uncertainty in the momentum (i.e. know position, not momentum). Conversely, a wave packet that is more spread out, giving more uncertainty in the position of the particle, will need fewer waves of different wavelengths to make the packet and will have less uncertainty in the momentum (i.e. don't know position, but know momentum). It turns out that a particle's exact position and momentum are constrained by the equation $\Delta x \Delta p = h/4\pi$, where the Δ indicates the uncertainty in the quantity. Therefore, the less the uncertainty in the position of the particle, the greater the uncertainty in the momentum and vice versa.

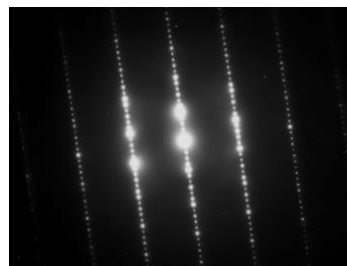


In classical motion, an object's future position and trajectory can be predicted exactly if the current position, velocity, and applied forces are known. However, the deterministic motion of classical particles does not apply to the quantum world where a particle exists as a smeared-out wave of likelihood until it is observed. This is not seen in our everyday lives because the quantum effects are only noticeable on the tiny, quantum scale. For example, the motion of an electron cannot be precisely modelled like a baseball moving through the air. The inability to predict exactly the motion of quantum objects (electrons, protons, photons, etc.) is a function of the probabilistic nature of quantum mechanics and not due to the lack of technology for measurements. The momentum (motion and position) of particles at small scales cannot both be known simultaneously because of the wave-like properties they exhibit on that scale and therefore requires quantum mechanics and the ability to use probability in describing how systems evolve over time.

The lesson begins with Heisenberg's uncertainty principle, which places a fundamental limit on the amount of information that can be known about a quantum object simultaneously. The activities surrounding this idea will revolve around how a photon's position and momentum cannot precisely be known at the same time. Students will use pencils and laser pointers of various wavelengths with a guided activity to make sure this concept is fully internalized. Student's will then be introduced the idea of a wavefunction, which is the conceptualization of a

quantum object's probabilistic nature. The activities included for the wavefunction will support the idea that the position of a quantum particle, while it has momentum, cannot fully be known without collapsing the wavefunction entirely (i.e. measurement). Measurements in quantum physics differ from classical measurements in that they are not passive, but rather they actively force the system to "choose" a single definite value thereby changing its state irreversibly. (e.g. a coin tossed in the air is both heads and tails until measured at which time it is either heads OR tails). Since measuring requires a physical interaction, the act of measuring alters the state of the system and the measurement often randomizes or scrambles the properties.

If light can behave as a wave, but can it also behave like a particle, can a particle behave as a wave if we get down to the quantum limit? French physicist Louis de Broglie proposed that this would be true, but without evidence. Using Planck's hypothesis about quanta, de Broglie argued that matter waves would have a wavelength given by $\lambda = h/p$, where p is the momentum of the particle. Because h is a very small number, the wavelengths are also very small, which is why we don't experience matter as a wave in everyday life. However, if one looks on the very small scale one should be able to see wave behavior. And that is what was found – electrons could produce a diffraction pattern on the scale of the de Broglie wavelength. The figure above shows an electron diffraction pattern (Britannica Editors. "electron diffraction". Encyclopedia Britannica, 27 Sept. 2016, <https://www.britannica.com/science/electron-diffraction>. Accessed 2 June 2026.)



Classically, particles have a definite position and momentum that can be identified at any time assuming enough is known about initial conditions. For quantum particles, the position and momentum cannot be known simultaneously. This is called Heisenberg's Uncertainty Principle (HUP) and it is the foundational idea in quantum physics. Students will struggle with this concept due to the abstract nature of this phenomenon, so the explore activities work to build conceptual understanding of a complicated idea. The first exploration is using a simulation that illustrates the complementary nature of position and momentum uncertainty needed to make a wave packet. The next step is to explore the idea with photons through the use of simple materials that can be found in any physics or chemistry classroom (lasers, pencils, and basic supplies).

ENGAGE 1 Simulation of a wave packet

Open the link <https://www.compadre.org/osp/EJSS/4164/192.htm>

Adjust the blue slider at the bottom of the graph. What is the relationship between momentum and the position of the particle?

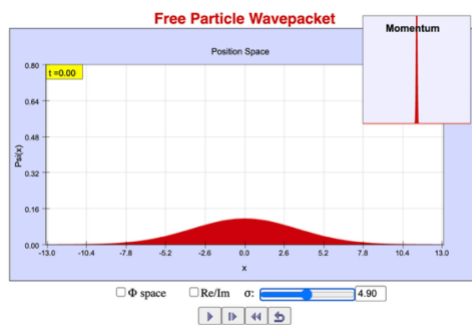
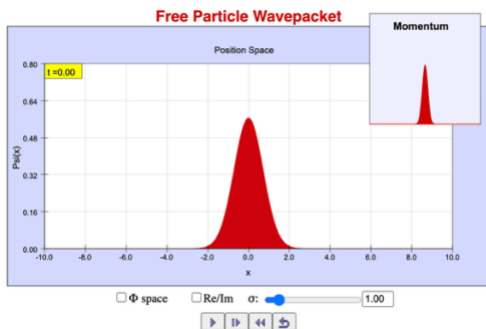
ANS: as the position in space is reduced (width of the wave packet or where the particle would be localized) the possible momentum is spread out and less known (as the position in space is wider, the momentum is more specific).



What conclusions can be made regarding position and momentum?

ANS: When we try to localize the position more, the uncertainty in the momentum grows because we have to use more wavelengths (momenta) to constrain the width of the wave packet, and vice versa. The uncertainties in the position and momentum are complementary and are constrained by the equation $\Delta x \Delta p = h/4\pi$.

Sample Graphs are below



ENGAGE 2

Dot size in diffraction and the HUP applied to photons

****Note: Laser safety is extremely important in this lesson since green and blue light will be used. Students must follow all guidelines for laser safety and make sure no laser light gets anywhere near any other student.**

The engage portion of this lesson allows students to see what happens when light moves through a single slit where the width of the slit can be varied. In groups, students will use two pencils held together by rubber bands (shown to the right) where they can squeeze the pencils together to create a smaller width. For this



Connect two pencils together with elastic bands.

Credit: Perimeter Institute



engage activity students can hold the laser directly against the pencils and make qualitative observations of the central maximum that shines on a screen of some kind. The easiest way to execute this safely is to have the students shine the laser through the pencils and onto the ceiling. If a whiteboard or other shiny surface is used, be careful of reflected light. Also, the greater distance between the slit and the surface allows students to see the pattern more clearly. Students should notice that the central maximum size increases with a smaller slit which is counterintuitive in nature and the reason this engage is effective for this lesson. To understand how the width of the central bright spot when a laser is shown through a slit is related to Heisenberg's Uncertainty Principle. It is imperative in this activity for students to think of the laser beam as being a string of photons coming out of the laser (over a quintillion in one pulse!) to look at this activity as supporting the particle nature of light where the particle's exact position and momentum are constrained by the equation $\Delta x \Delta p = h/4\pi$.

1. When the laser shines through the wide slit, it makes a pattern with a central maximum visible, how do you think the size of that central maximum will change when the width of the slit

is decreased (make a prediction)? *ANS: Student predictions will vary but they often say that with a narrower slit they will expect a narrower central maximum*

2. When the width through which the laser is shown decreases what happens to the central maximum size?

ANS: The central maximum size should increase with a decrease in slit width

3. After the pencils are rotated, what happens to the pattern?

ANS: The pattern of the central maximum and other fringes simply rotates with the rotation of the pencils. This part of the engage is done so that students can better understand the next activity and the explanation of this phenomenon.

Engage 2: Student Instructions

Attach 2 pencils together with rubber bands so there is a variable gap between them.

Predict the pattern that would result from light passing in the slit between them.

Test your prediction using a red laser and shining the laser at the wider part of the gap as well as the smaller part.

Explain the pattern(s) you observed and justify whether it supports the wave or particle model for light.

EXPLORE

Where or How Fast?

Objective: Discover the relationship between photon momentum and the Heisenberg Uncertainty Principle

Materials: Two pencils connected by a rubber band, red, green and violet lasers, white board (for a screen), markers, ruler.

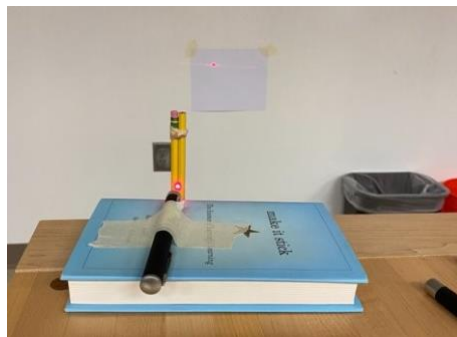
Procedure:

1. Take the pencil set up you used in the engage portion of the lesson and find a way to standardize the distance from the pencil to the screen (see picture). It is very important to keep this distance the same for every trial.

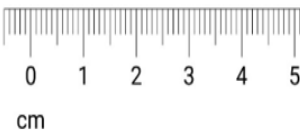
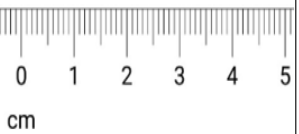
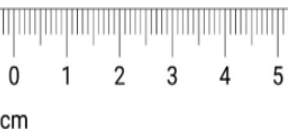
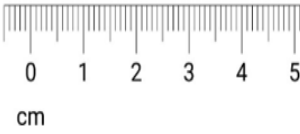
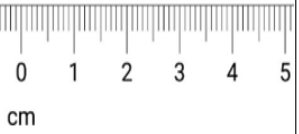
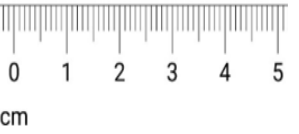
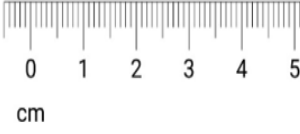
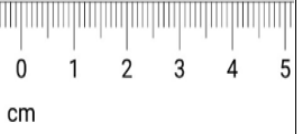
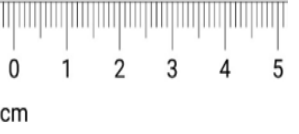
2. Shine the red laser through the pencils and toward the screen through the widest slit in the pencils, then the medium sized slit, and finally through the narrow slit. The widest slit is generally towards the top of the pencils, medium towards the middle and narrow towards the bottom (furthest from the rubber bands).

3. Carefully mark the width of the central spot/smear (i.e., central maximum) created by the laser through each slit width on the data sheet.

4. Repeat steps 2 and 3 using the green and violet lasers.



Data collection

	Red	Green	Violet
Wide Slit			
Medium Slit			
Narrow Slit			

What general relationships do you observe in this experiment? Relate the central maximum size to slit width and wavelength

Analysis questions:

Quantum physics says that we can model the laser light from Part 1 as a stream of individual particles called photons. A complementary relationship similar to the one that was found between the width of the slit and the spread of the central spot exists for *individual photons*.

Figure 1 is an image of one photon (represented by the wavy line) passing through the slit in the experiment from Part 1 from a side-view perspective. The width of the slit is d . The grey oval on the wall represents the spread of the central diffraction pattern.

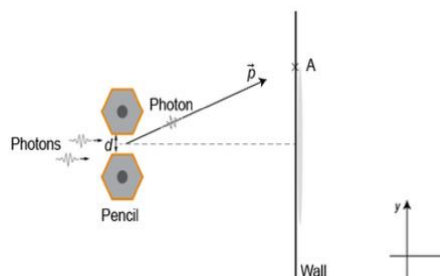


Figure 1 A photon in the single-slit experiment

When a photon passes through the slit in **Figure 1**, we know that it is somewhere between the edges of the slit. Physicists say that the *uncertainty* in the photon's position y is equal to the width of the slit.

They use the symbol Δ for uncertainty, so Δy is the uncertainty in the photon's position along the y -axis where $\Delta y = d$.

1. What did you do to change Δy ?

ANS: After passing through the slit, the photon travels toward the wall with a momentum $\vec{p}$. Quantum physics says that most photons will hit the wall at a random location within the central spot, so we cannot predict exactly where each photon will land.

2. For the photon in **Figure 1**, resolve the momentum into x and y components (i.e.,

- p_x and p_y). Which component is associated with the size of the diffraction pattern?
3. The uncertainty Δp_y is equal to the size of the range of possible p_y values.
Describe how Δp_y changes when you make the diffraction pattern taller.
 4. When you decreased d , did Δp_y increase or decrease? Explain your answer.
 5. Describe the relationship between the complementary properties Δp_y and Δy .

The mathematical relationship $\Delta x \Delta p \geq h/(4\pi)$, where $h = 6.63 \times 10^{-34}$ J·s is Planck's constant, is called Heisenberg's uncertainty principle. It describes a fundamental limit to how precisely we can simultaneously know the position and momentum of a photon. Quantum physics says that it also applies to all subatomic particles. Although we arrived at it by thinking about the single-slit experiment, it also holds in all other experiments.

How might this fundamental limit affect our ability to reduce computer chips to even smaller scales?

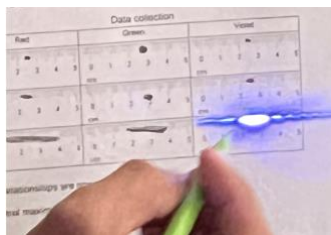
Teacher Instructions:

1. Be sure students are following all safety protocols and use a setup that does not cross the paths of any other set up so that students do not unintentionally shine laser light at another student. Green and violet light are very harmful if shined in the eyes and this point needs to be made clearly to all students. It is best to limit the number of green and violet lasers so they can be carefully monitored.
2. Ensure that students are keeping the distance from the pencils to the screen constant as any changes to that variable will interfere with the data they need to be taking.
3. The primary function of this lab is to get students to understand the difference between the size of the bright spot made by red, green, and violet lasers. Students should see that for each slit width, red will produce the widest bright spot, and the bright spot width will decrease as the wavelength decreases.
4. Students must be diligent about how they are measuring the width of the spot because if they are not careful they will get data that goes against the concepts of the lab that are trying to be conveyed. If students are not seeing data with enough variance, have them increase the distance between their pencils and the screen.
5. The lesson begins with gaining a solid understanding of the Heisenberg Uncertainty principle where the narrowing of the pencil gap increases the spread of the central maximum that forms on the screen from the laser. It needs to be explicitly stated by the instructor that the increase in the smear is the increase in the momentum uncertainty. The uncertainty in momentum increases when the pencil gaps narrow because when the gap is more narrow, more is known about the position which means less must be known about the momentum.

Sample Data

	Red	Green	Violet
Wide Slit			
Medium Slit			
Narrow Slit			

Central maxima

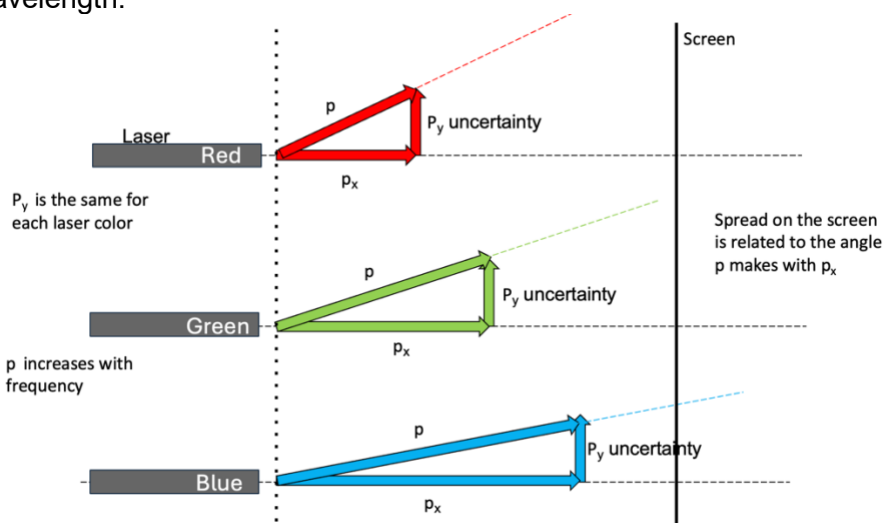


	Red $\lambda = 656 \text{ nm}$	Green $\lambda = 532 \text{ nm}$	Violet $\lambda = 405 \text{ nm}$
Wide Slit			
Medium Slit			
Narrow Slit			

EXPLAIN

Have students view <https://youtu.be/a8FTq2qMutA>

- $\Delta x \Delta p \geq h/(4\pi)$ is the mathematical relationship called Heisenberg's Uncertainty Principle which posits that as certainty in an object's position increases, the certainty in the momentum decreases
- Δx is the uncertainty in the position and Δp is the uncertainty in the momentum. $h/(4\pi)$ is a very small number so wave phenomena do not show up in the macro world.
- de Broglie's equation $p = h/\lambda$ relates the wavelength of the matter wave to the momentum, but the effects of matter waves can only be seen on the quantum scale of the de Broglie wavelength.



- For the same slit width, red light (long wavelength, low momentum) produces a larger diffraction smear, while violet light (short wavelength, high momentum) produces a smaller smear.

- A shorter wavelength means there is greater momentum in the photons, but the overall uncertainty in position (Δy) stays the same for the same slit width between the pencils. This leads to a larger (in width) maxima with red (longer wavelength, lower momentum) than with violet (smaller wavelength, greater momentum) for the same slit width.



The smaller the wavelength of the light, the smaller the central maximum on the screen. This means that for the same slit width, red will have the largest central maximum, and the central maximum will decrease when green is shown and be the smallest with blue/violet.

Students will observe the width of the central maximum as a smearing of the laser light across the page. The central maximum can be thought of as all the individual photons striking the screen in random locations while following the Heisenberg uncertainty principle.

Sample data for red at varying slit widths with the top being the narrow slit is shown to the right.

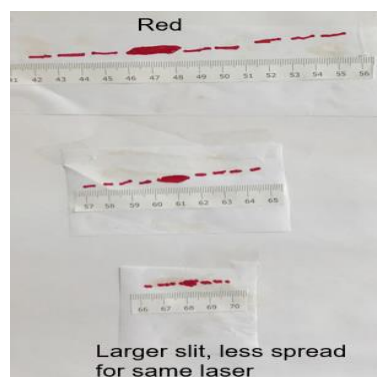
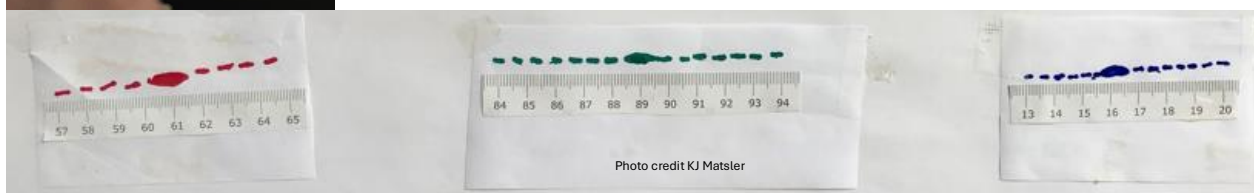


Photo credit KJ Matsler

Below is the data for different lasers at the narrow slit. It is more difficult to see, but the central maximum spread should be the widest for red and decrease in size with a decrease in wavelength.



EXTEND

Applications for Heisenberg Uncertainty Principle include many modern technologies for imaging, computing or precision measurements. Consider having students research applications such as:

Electron Microscopes—high-resolution images that constrain the position of electrons.

Magnetic Resonance Imaging—relies on uncertainty relations between energy and time.

Gravitational Wave Detection—uses “squeezing” to precisely measure distances.

Quantum Tunneling—designing of flash memory, transistors, and studying stellar nuclear fusion in the sun.

Lasers—limit of laser pulse is governed by uncertainty principle.